rearrange eq: let $z = \cos\theta$ $\dot{z} = -\sin\theta\,\dot{\theta}$

$$\frac{1}{2}\frac{P_\phi^2(1-\cos\theta)^2}{I_1 \sin^2\theta} + \frac{1}{2}I_1\dot{\theta}^2 + Mgh\cos\theta = Mgh$$

$$\frac{1}{2}\frac{P_\phi^2(1-z)^2}{I_1 z^2 \dot{\theta}^2} + \frac{1}{2}I_1\dot{\theta}^2 + Mghz = Mgh$$

$$\frac{P_\phi^2}{I_1}\frac{(1-z)^2}{z^2\dot{\theta}^2} = Mgh(1-z) - \frac{1}{2}I_1\dot{\theta}^2$$